ME 321: Fluid Mechanics-I

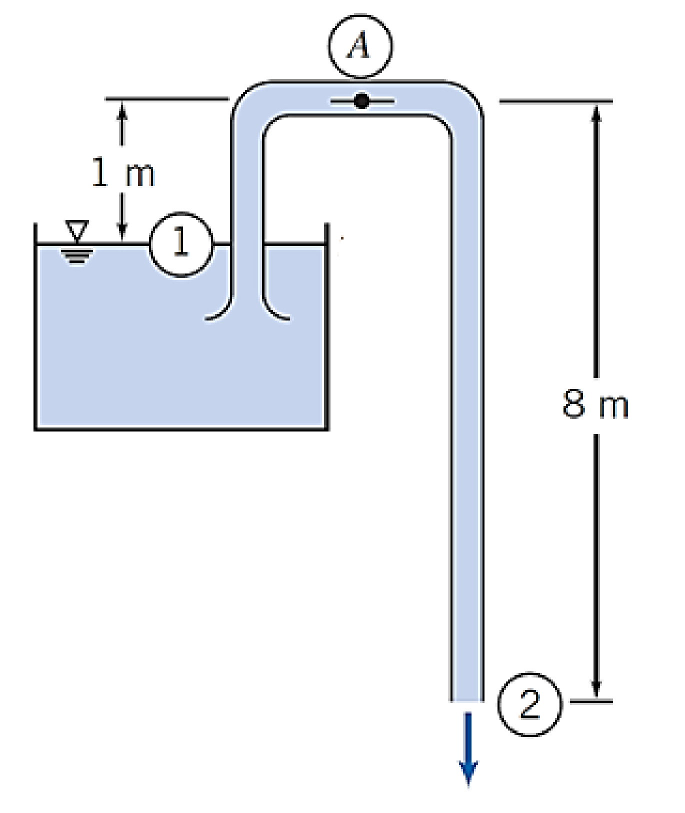
Prof. Dr. A.B.M. Toufique Hasan
Department of Mechanical Engineering
Bangladesh University of Engineering and Technology (BUET)

Lecture - 10 (05/07/2025)
Fluid Dynamics: Applications of Bernoulli Equation

toufiquehasan.buet.ac.bd
toufiquehasan@me.buet.ac.bd

Problem # 8

A U-tube acts as a water siphon. The bend in the tube is 1 m above the water surface; the tube outlet is 7 m below the water surface. The water issues from the bottom of the siphon as a free jet at atmospheric pressure. Determine (after listing the necessary assumptions) the speed of the free jet and the minimum absolute pressure of the water in the bend.



Solution:

Assumptions:
(1) Neglect friction.
(2) Steady flow.
(3) Incompressible flow. Irrotational flow
(4) Flow along a streamline.
(5) Reservoir is large compared with pipe.

Apply the Bernoulli equation between points ① and ②.

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Since $\text{area}_{\text{reservoir}} \gg \text{area}_{\text{pipe}}$, then $V_1 \approx 0$. Also $p_1 = p_2 = p_{atm}$, so

$$gz_1 = \frac{V_2^2}{2} + gz_2 \qquad \text{and} \qquad V_2^2 = 2g(z_1 - z_2)$$

11.7 m/s ⟵

Problem # 8

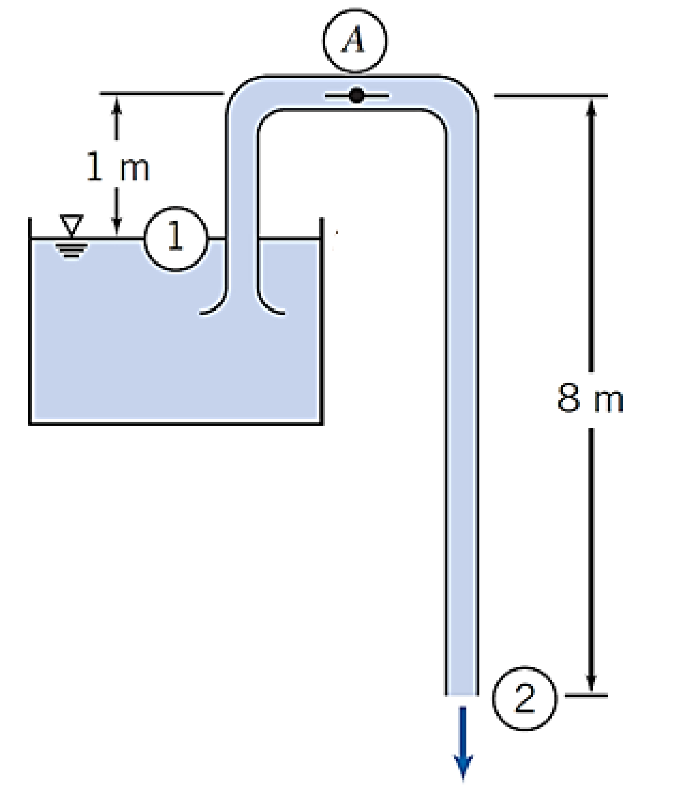
To determine the pressure at location Ⓐ, we write the Bernoulli equation between ① and Ⓐ.

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A$$

Again $V_1 \approx 0$ and from conservation of mass $V_A = V_2$. Hence

$$\frac{p_A}{\rho} = \frac{p_1}{\rho} + gz_1 - \frac{V_2^2}{2} - gz_A = \frac{p_1}{\rho} + g(z_1 - z_A) - \frac{V_2^2}{2}$$

$$p_A = 22.8 \text{ kPa (abs) or } -78.5 \text{ kPa (gage)} \longleftarrow$$



Real flow system

Modification of Bernoulli equation is a must for **real flow systems**:

Real flow systems must account for loss of energy, which is frequently known as **head loss**.

1) **Major loss** (due to viscous effect /fluid friction /viscosity)
2) **Minor loss / local losses** (due to different pipe fittings, etc.)

Details in ME 323

Modified Bernoulli relation comes as:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

(1) is the upstream point and
(2) is the downstream point

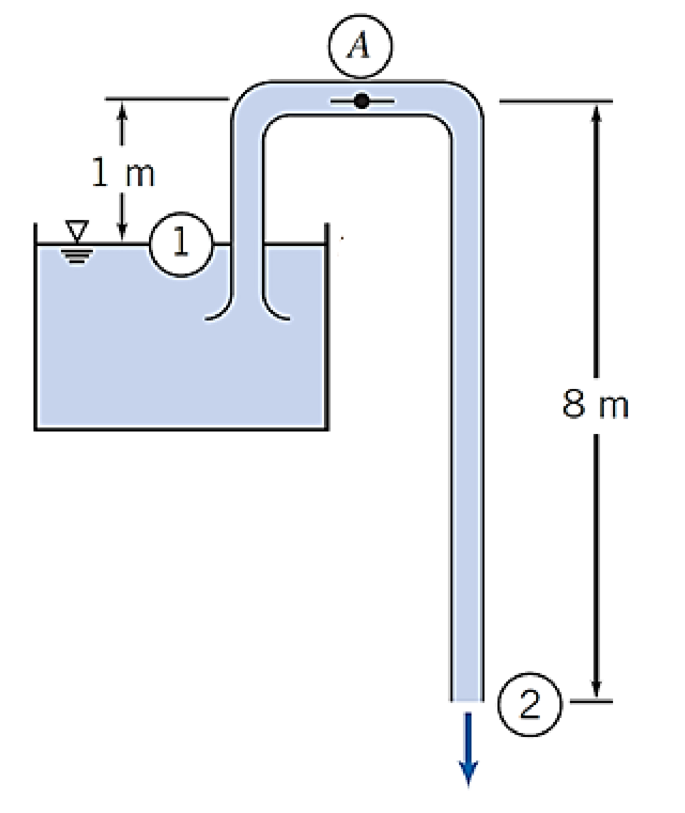
h_L = head loss

$$h_L = \sum f \frac{L}{D}\frac{v^2}{2g} + \sum k \frac{v^2}{2g}$$

Problem # 9

A U-tube acts as a water siphon. The bend in the tube is 1 m above the water surface; the tube outlet is 7 m below the water surface. The water issues from the bottom of the siphon as a free jet at atmospheric pressure. Determine (after listing the necessary assumptions) the speed of the free jet and the minimum absolute pressure of the water in the bend if the head loss count is

$$h_L \approx 2.3\,\frac{V^2}{2g}$$

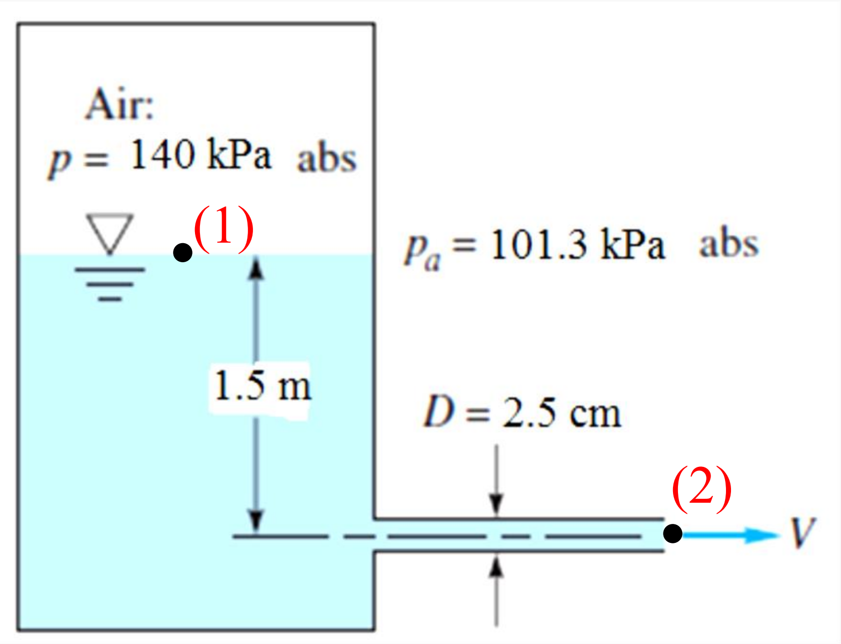


Problem # 10

The liquid in the figure below is kerosene (SG = 0.8). Estimate the flow rate from the tank for

(a) No losses and

(b) Pipe losses $h_L \approx 4.5\dfrac{V^2}{2g}$



Solution:

(a) $$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

$$\Rightarrow \frac{140\times10^3}{\gamma}+\frac{0^2}{2g}+1.5=\frac{101.3\times10^3}{\gamma}+\frac{V_2^2}{2g}+0$$

$$\Rightarrow V_2 = \qquad (\equiv V)$$

$$\therefore \; Q = AV = ?$$

(b) $$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

$$\Rightarrow \frac{140\times10^3}{\gamma}+\frac{0^2}{2g}+1.5=\frac{101.3\times10^3}{\gamma}+\frac{V_2^2}{2g}+0+4.5\frac{V_2^2}{2g}$$

$$\Rightarrow V_2 = \qquad (\equiv V)$$

$$\therefore \; Q = AV = ?$$

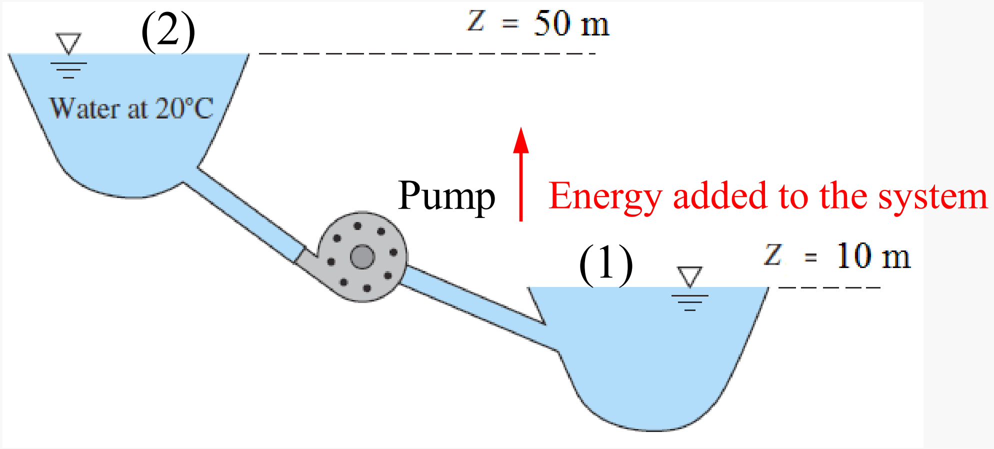
Flow system with turbomachinery

Modified Bernoulli equation i.e. the **energy equation** in a **flow system with pump**:

No head loss: $$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 \quad + h_P \quad = \frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

h_P = head (energy) added to the system

$$P_{pump} = \gamma Q h_P$$ **(pump hydraulic power)**



Pump:
Input: Electrical power (in general)
Output: hydraulic power

With head loss: $$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 \quad + h_P \quad = \frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad + h_L$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\gamma Q h_P}{P_{in}\text{ (elect. power)}}$$

h_L = head loss (major/minor) to be added at downstream

$$h_L = K\frac{V^2}{2g}$$

Flow system with turbomachinery

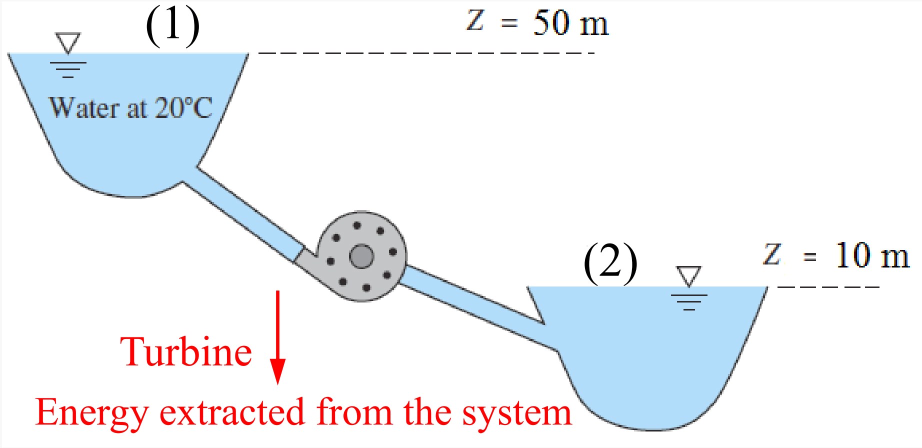
Modified Bernoulli equation i.e. the **energy equation** in a **flow system with turbine**:

No head loss:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_T$$

h_T = head (energy) extracted from the system

$P_{turbine}=\gamma Q h_T$ **(turbine hydraulic power)**



Turbine:

Input: hydraulic/mechanical power
Output: Electrical power

$$\eta=\frac{P_{out}}{P_{in}}=\frac{P_{in}\,(\text{elect. power})}{\gamma Q h_T}$$

With head loss:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_T+h_L$$

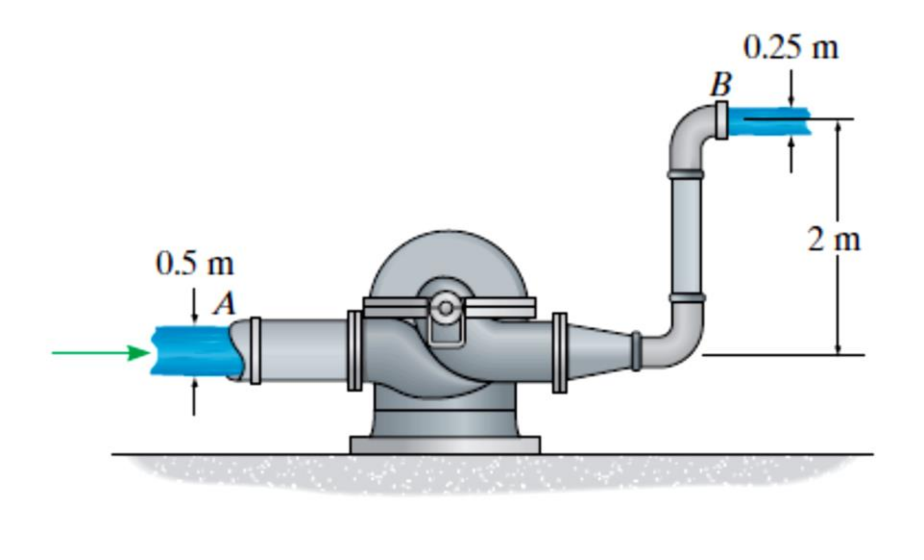
h_L = head loss (major/minor) to be added at downstream

Problem 11

The electrical power input to the pump is 10 kW. If the pump has an efficiency of 80%, and the increase in pressure from ***A*** to ***B*** is 100 kPa, determine the volumetric flow rate of water through the pump in cases of

(i) No head loss between A to B

(ii) Head loss between A to B is 1.25 m.



Solution:

(i) <u>No head loss between A to B:</u>

Bernoulli equation between points A and B for this case is-

$$\frac{p_A}{\gamma}+\frac{V_A^2}{2g}+z_A+h_p=\frac{p_B}{\gamma}+\frac{V_B^2}{2g}+z_B \qquad (i)$$

; h_p is the head developed by the pump

From continuity equation:

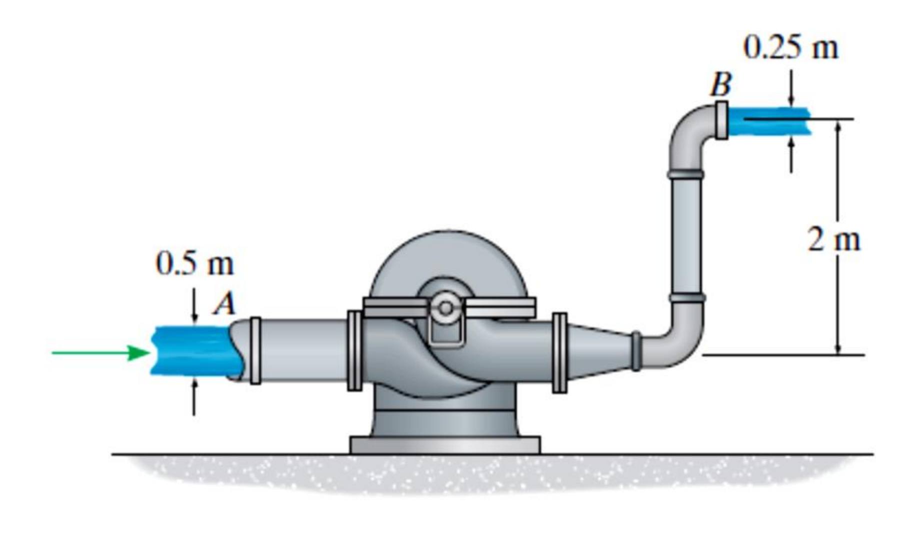
$$Q=v_A A_A=v_B A_B \qquad \text{(unknown)}$$

$$\Rightarrow Q=v_A\left(\frac{\pi}{4}d_A^2\right)=v_B\left(\frac{\pi}{4}d_B^2\right)$$

$$\Rightarrow Q=v_A\left(\frac{\pi}{4}0.5^2\right)=v_B\left(\frac{\pi}{4}0.25^2\right)$$

$$\Rightarrow v_A=5.09Q \quad \& \quad v_B=20.37Q$$

Problem 11



For the pump

$$\eta = \frac{P_{out}}{P_{in}} \left(\frac{\text{Hydraulic power output}}{\text{Electrical power input}} \right)$$

$$\Rightarrow 0.8 = \frac{P_{out}}{10\times10^3}$$

$$\Rightarrow P_{out} = 0.8\times10\times10^3$$

$$\Rightarrow \gamma Q h_P = 0.8\times10\times10^3$$

$$\Rightarrow h_P = \frac{0.8\times10\times10^3}{\gamma Q}$$

$$\boxed{\Rightarrow h_p = \frac{0.8155}{Q}}$$

Now consider equation (i)

$$\frac{p_A}{\gamma} + \frac{V_A^{\,2}}{2g} + z_A + h_P = \frac{p_B}{\gamma} + \frac{V_B^{\,2}}{2g} + z_B \qquad (i)$$

$$\Rightarrow \frac{V_A^{\,2}}{2g} + 0 + h_P = \frac{p_B - p_A}{\gamma} + \frac{V_B^{\,2}}{2g} + 2$$

$$\Rightarrow \frac{(5.09Q)^2}{2g} + 0 + \frac{0.8155}{Q} = \frac{100\times10^3}{(1000\times9.81)} + \frac{(20.37Q)^2}{2g} + 2$$

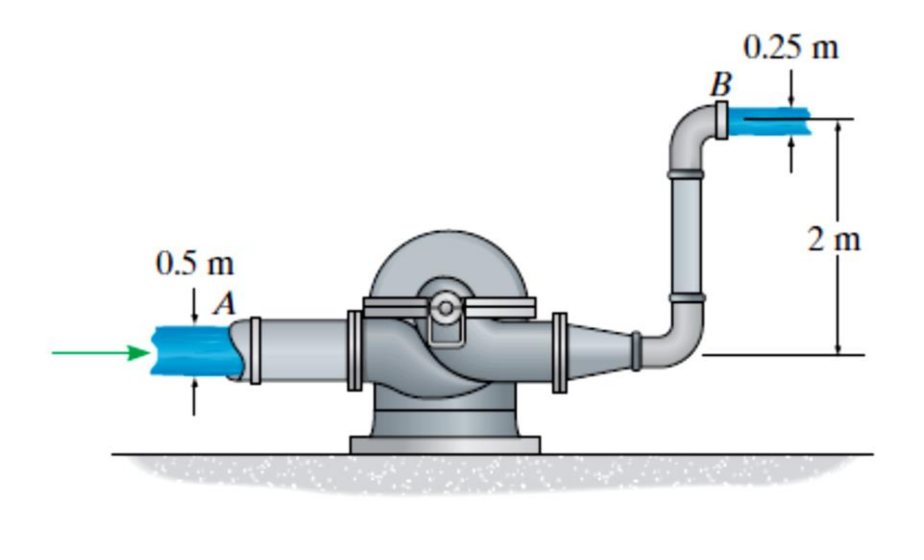
$$\boxed{\Rightarrow \frac{0.8155}{Q} = 12.19 + 19.83Q^2}$$

Problem 11

On solving the last equation to get the flow rate, Q: (***through numerical solution***)

$$\Rightarrow \frac{0.8155}{Q} = 12.19 + 19.83Q^2$$

$$\boxed{\Rightarrow Q \approx 0.0664 \text{ m}^3/\text{s} \quad (\equiv 239\text{m}^3/\text{hr}, 66.4\text{l/s})}$$ **Ans. (i)**



(ii) Head loss between A to B is 1.25 m:

Bernoulli equation between points A and B for this case is-

$$\frac{p_A}{\gamma} + \frac{V_A^{\,2}}{2g} + z_A + h_P = \frac{p_B}{\gamma} + \frac{V_B^{\,2}}{2g} + z_B + h_L \qquad (ii)$$

; h_p **is the head developed by the pump**
h_L **is the head loss from points A to B**

$$\Rightarrow \frac{V_A^{\,2}}{2g} + 0 + h_P = \frac{p_B - p_A}{\gamma} + \frac{V_B^{\,2}}{2g} + 2 + 1.25$$

$$\Rightarrow \frac{(5.09Q)^2}{2g} + 0 + \frac{0.8155}{Q} = \frac{100\times 10^3}{(1000\times 9.81)} + \frac{(20.37Q)^2}{2g} + 3.25$$

$$\boxed{\Rightarrow \frac{0.8155}{Q} = 13.44 + 19.83Q^2}$$

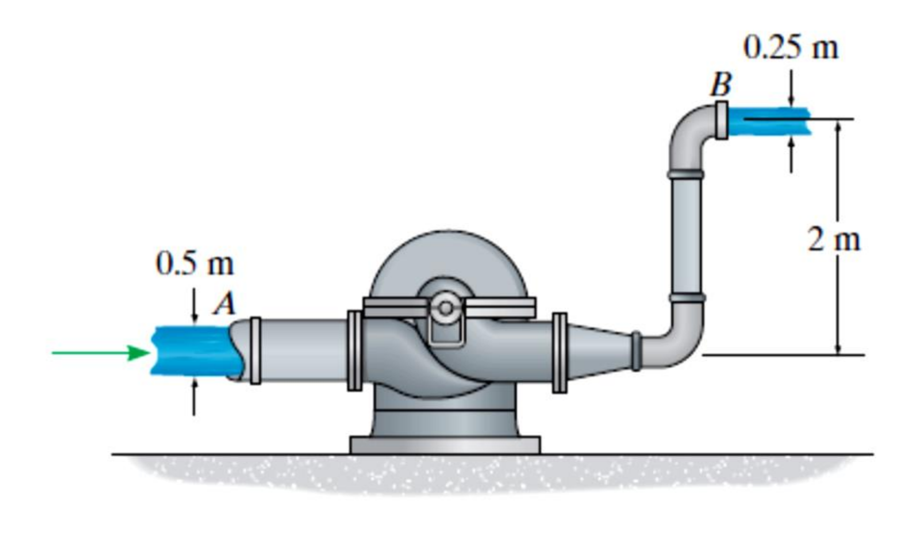
Problem 11

On solving the last equation to get the flow rate, Q: (***through numerical solution***)

$$\Rightarrow \frac{0.8155}{Q} = 13.44 + 19.83Q^2$$

$$\boxed{\Rightarrow Q \approx 0.0604\ \text{m}^3/\text{s} \quad (\equiv 218\text{m}^3/\text{hr},\ 60.4\ \text{l/s})}$$

Ans. (ii)

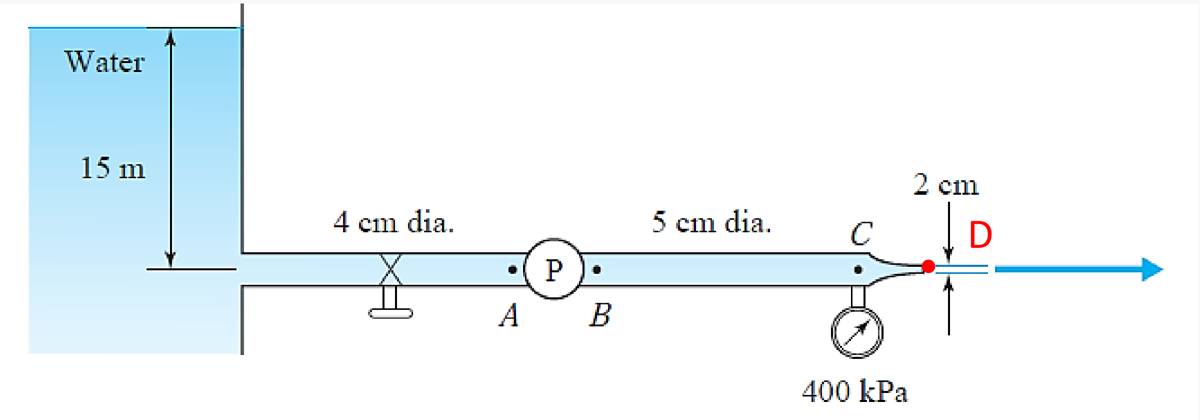


Volumetric flow rate will be reduced in case of head loss due to fluid friction (major loss) and pipe fittings (minor loss).

Head losses will be covered in detail in ME 323 (L3 T2)

Problem 12

Find the power requirement of the 85%-efficient pump shown in Fig. if the loss coefficient up to *A* is 3.2, and from *B* to *C*, *K*=1.5. Neglect the losses through the exit nozzle.
Also, calculate p_A and p_B.



$$h_L = K\frac{V^2}{2g}$$

Solution:

$$Q_C = Q_D$$

$$\frac{\pi}{4}0.05^2\, V_C = \frac{\pi}{4}0.02^2\, V_D$$

$$\therefore Vc = 0.16\, VD$$

Bernoulli equation between points C and D (across the nozzle) -

$$\frac{p_C}{\gamma} + \frac{{V_C}^2}{2g} + z_C = \frac{p_D}{\gamma} + \frac{{V_D}^2}{2g} + z_D + h_{L\,C-D}$$

$$\frac{400\times 10^3}{9810} + \frac{(0.16V_D)^2}{2g} + 0 = 0 + \frac{{V_D}^2}{2g} + 0 + 0$$

$; h_{L\,C-D} = 0$ *(no loss through the nozzle)*

$$\therefore VD = 28.6\, m/s \qquad \therefore VC = 4.6\, m/s$$

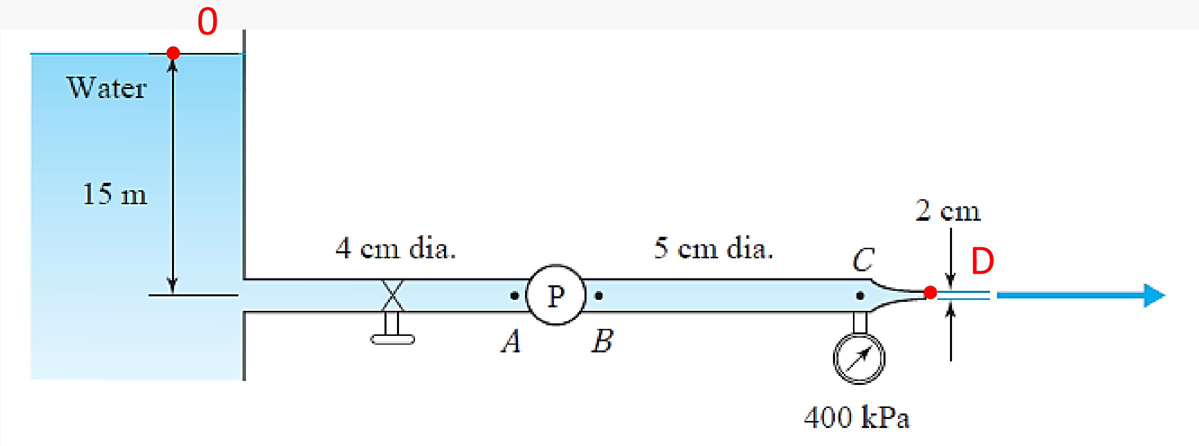
Problem 12

Considering points B and C -

$$Q_B = Q_C$$
$$\therefore VB = VC = 4.6\, m/s$$

Considering points A and C -

$$Q_A = Q_C$$
$$\frac{\pi}{4} 0.04^2\, V_A = \frac{\pi}{4} 0.05^2 (4.6)$$
$$\therefore VA = 7.2\, m/s$$



Bernoulli equation between points 0 and D (surface to exit) -

$$\frac{p_0}{\gamma} + \frac{{V_0}^2}{2g} + z_0 + hp = \frac{p_D}{\gamma} + \frac{{V_D}^2}{2g} + z_D + h_{L\,0-D}$$

$$0 + 0 + 15 + hp = 0 + \frac{28.6^2}{2g} + 0 + 3.2\frac{7.2^2}{2g} + 1.5\frac{4.6^2}{2g}$$

$$; h_{L\,0-D} = h_{L\,0-A} + h_{L\,B-D} = KA\frac{V_A^2}{2g} + K_B\frac{V_B^2}{2g}$$

$$\therefore hp = 36.8\, m$$

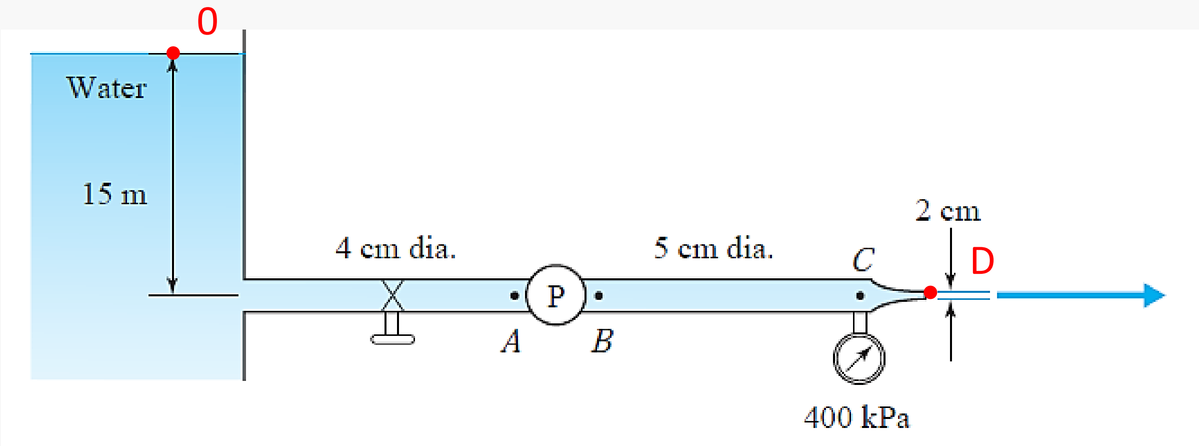
Problem 12

Pump requirement to run the pump

$$P_{elect.} = \frac{\gamma Q h P}{\eta}$$

$$= \frac{(9810)(\frac{\pi}{4}\times 0.02^2\times 28.6)(36.8)}{0.85}$$

= 3.82 kW (Ans.)



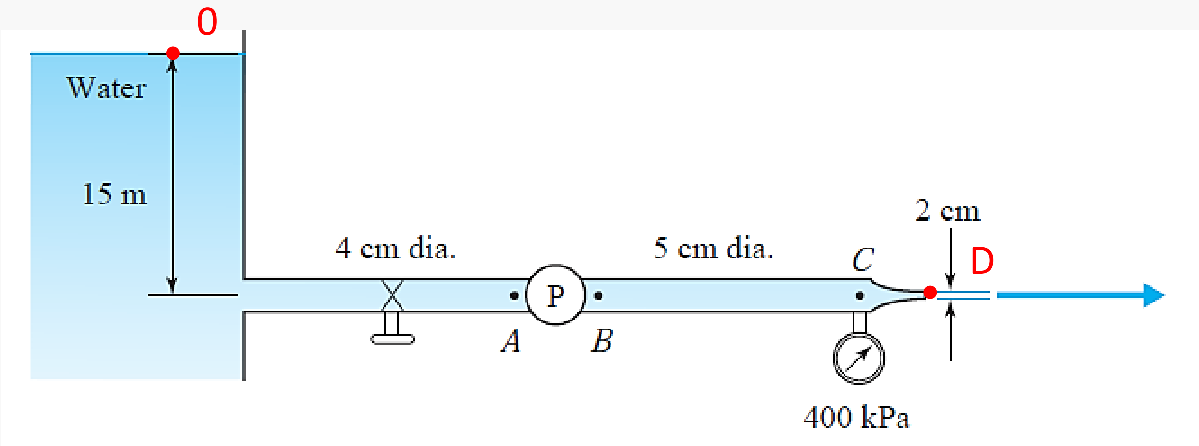
Bernoulli equation between points 0 and A (surface to A) -

$$\frac{p_0}{\gamma} + \frac{V_0^{\,2}}{2g} + z_0 = \frac{p_A}{\gamma} + \frac{V_A^{\,2}}{2g} + z_A + h_{L\,0-A}$$

$$0 + 0 + 15 = \frac{p_A}{\gamma} + \frac{7.2^2}{2g} + 0 + 3.2\frac{7.2^2}{2g}$$

$$\therefore pA = 38.3\ kPa\ (Ans.)$$

Problem 12



Bernoulli equation between points 0 and B (surface to B) -

$$\frac{p_0}{\gamma} + \frac{{V_0}^2}{2g} + z_0 + hp = \frac{p_B}{\gamma} + \frac{{V_B}^2}{2g} + z_B + h_{L\,0-B}$$

$$0 + 0 + 15 + 36.8 = \frac{p_B}{\gamma} + \frac{4.6^2}{2g} + 0 + 3.2\frac{7.2^2}{2g}$$

$$; h_{L\,0-B} = h_{L\,0-A} + h_{L\,A-B} = KA\frac{V_A^2}{2g} + 0$$

$$\therefore pB = 414.6\ kPa\ (Ans.)$$